Statistical Signal Processing

Minor 2: (19.3.2016)

Time: 1 hour Max. Marks: 25

1. Design an optimal causal Linear predictor for a process $x(n)$ that has a power spectrum

$$P_x(z) = \frac{(1 - 0.6z^{-1} + 0.36z^{-2})(1 - 0.6z + 0.36z^2)}{(1 - 0.8z^{-1})(1 - 0.8z)}$$

Write a recursive (difference) equation for the predictor, and find the value of the mean square prediction error. (8)

2. Consider the design of an optimum smoothing filter for estimating a process $d(n)$ from the measurements

$$x(n) = d(n) + v(n)$$

Goal is to find a non-causal FIR filter that has a system function of the form:

$$W(z) = \sum_{k=-p}^{p} w(k) z^{-k}$$

to produce an estimate of the form

$$\hat{d}(n) = \sum_{k=-p}^{p} w(k) x(n-k)$$

(a) Derive the Wiener Hopf equation that minimizes the mean square error. (2)

(b) How would the Wiener Hopf Equations change if we need a causal filer with the same number of coefficients? (2)

(c) Qualitatively, when should we prefer the non-causal filter and vice-versa: For what kind of signal and/or noise, the causal filter will be superior to the non-causal filter? (2)

(d) FIR digital filters with linear phase (or zero phase) are important where frequency dispersion due to nonlinear phase is harmful. An FIR filter with zero phase is characterised by the property that $w(n) = w(-n)$. Thus the system function may be written as

$$W(z) = w(0) + \sum_{k=1}^{p} w(k)[z^{-k} + z^{k}]$$

Derive the Wiener Hopf Equations that define the optimum zero phase smoothing filter. (3)

3. Obtain a realization of the third order FIR filter in the form of a lattice structure:

$$H(z) = 1 + 0.5z^{-1} - 0.1z^{-2} - 0.5z^{-3}.$$

(8)